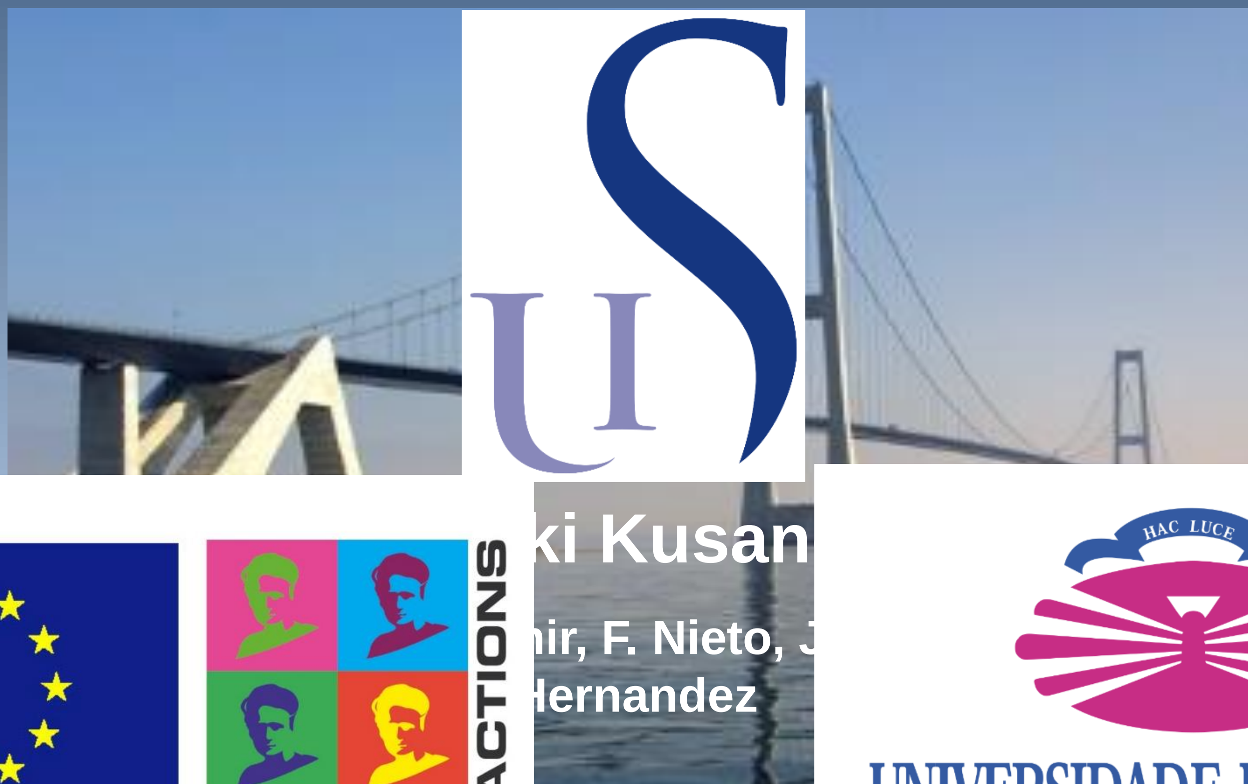
Reliability Based Design:

Optimization of Suspension Bridges with Emphasis on Aerodynamic Stability



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MARIE CURIE ACTIONS

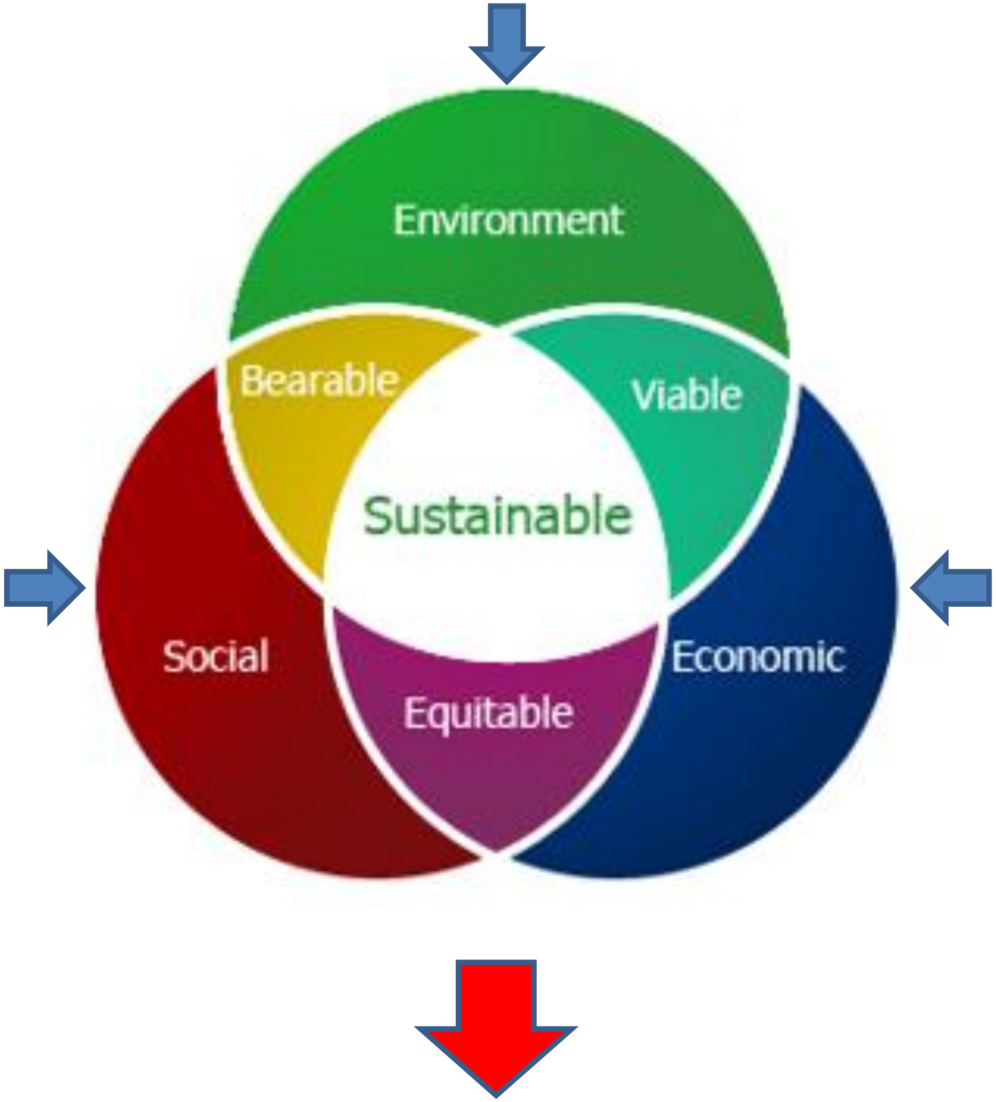
Outline

1. Motivation
2. Reliability Based Design Optimization
3. Reliability analysis of flutter
4. Application example
5. Summary

Structural Optimization

- Widely Used Technique (*e.g., aerospace, automobile, defense*)
- But Not So Common in Civil Engineering
- Many Uncertainties
- Reliability Based Design Optimization Considers Uncertainty Parameters Explicitly

Benefits & Payoffs



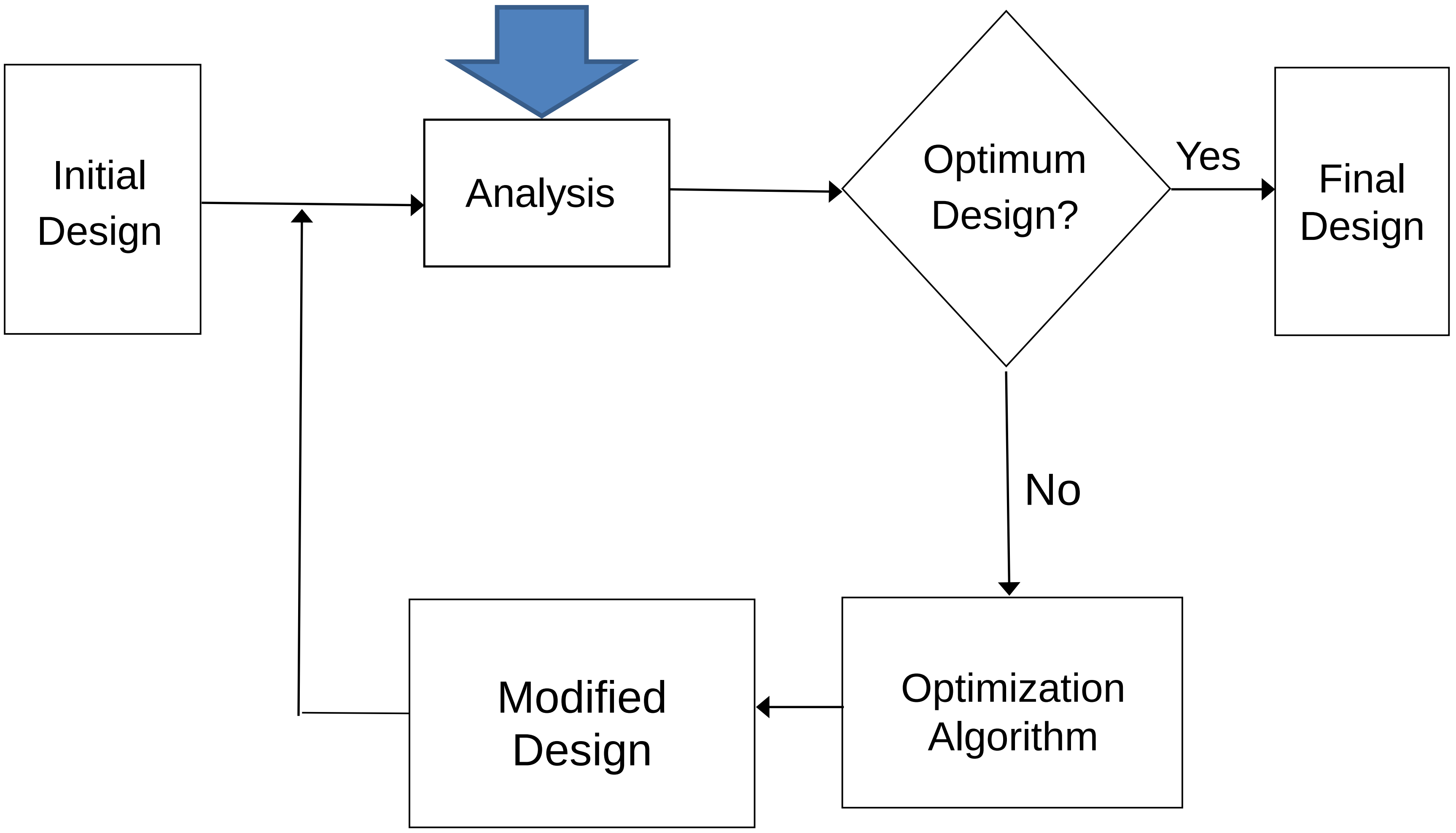
Robust Optimum Design + Reduce Carbon Emission

What Is Optimization?

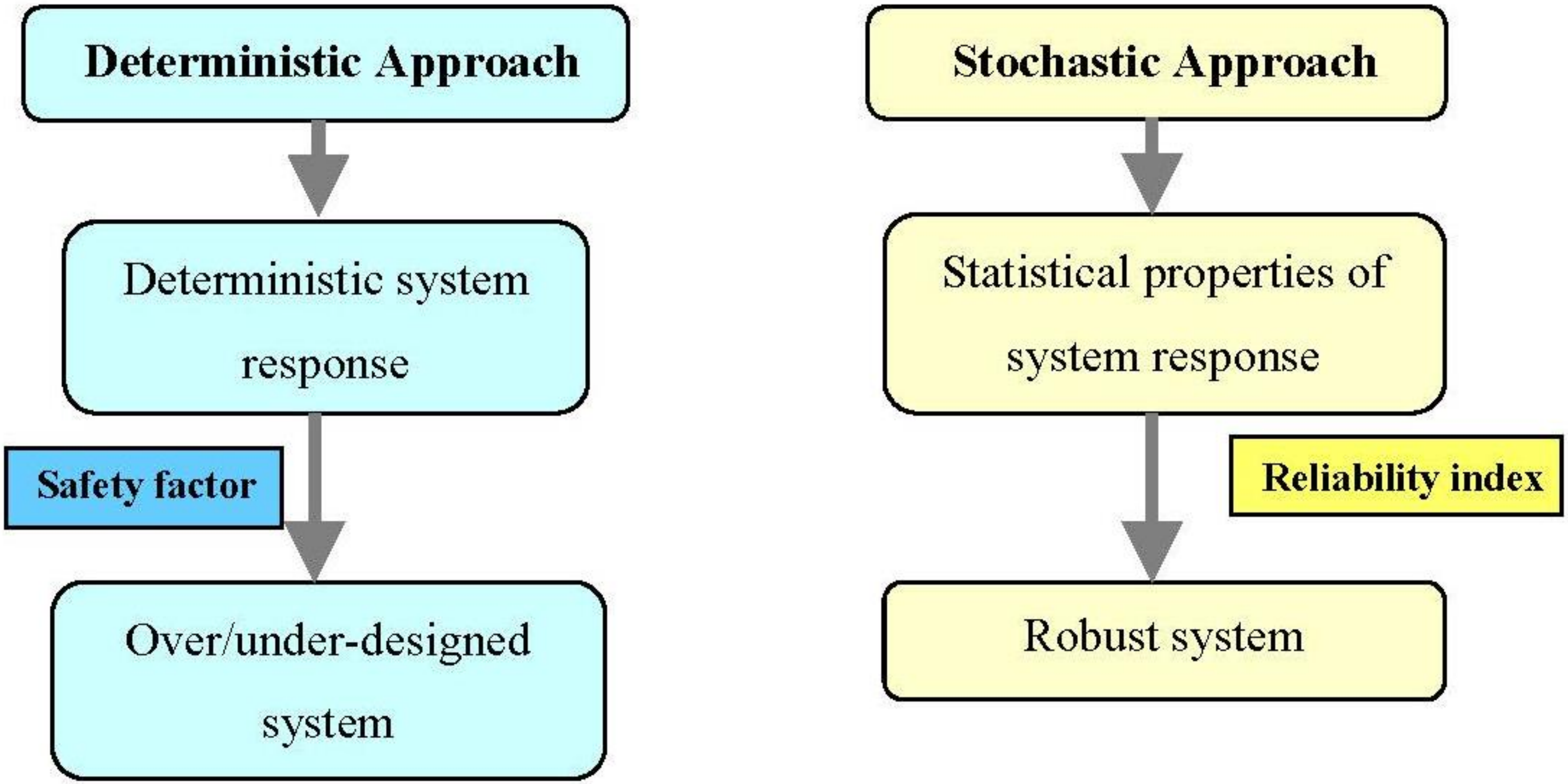
Design variable →	$\mathbf{x}=(x_1, x_2, \ldots, x_n)$
Objective function →	minimize $f(\mathbf{x})$
	such that
Side limits →	$lb_i \leq x_i \leq ub_i \quad i=1,2,\ldots,n$
	and
Constraints →	$g_1(\mathbf{x}) \leq b_1$
	$g_2(\mathbf{x}) \leq b_2$
	$\ldots$
	$g_m(\mathbf{x}) \leq b_m$

General Optimization Flow Chart

Uncertainty in Analysis?



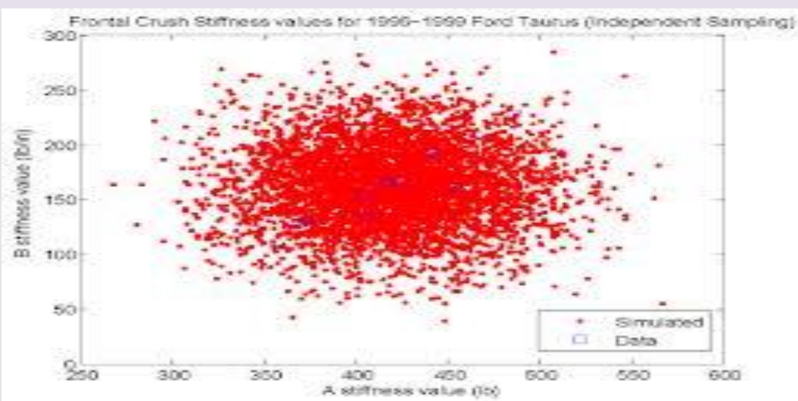
Uncertainty in Parameters



Two Methods

Sampling Methods

- Monte Carlo Sampling
- Latin Hypercube Sampling
- Importance Sampling



Moment Methods

- 1st Order Reliability Method
- 2nd Order Reliability Method

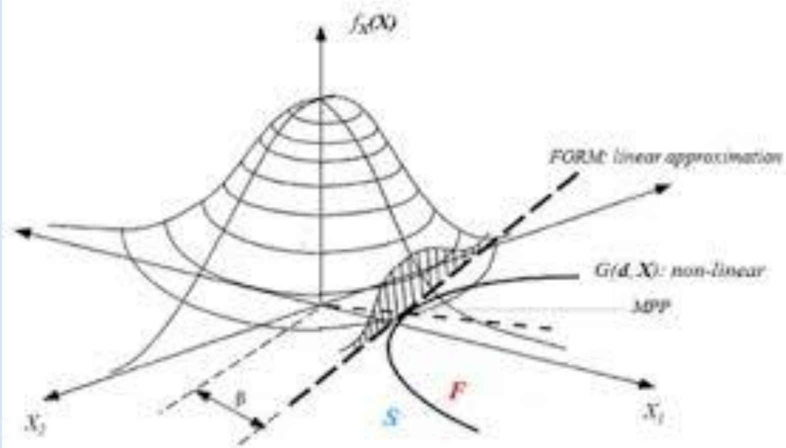


Figure 2. Illustration of the FORM approximation.

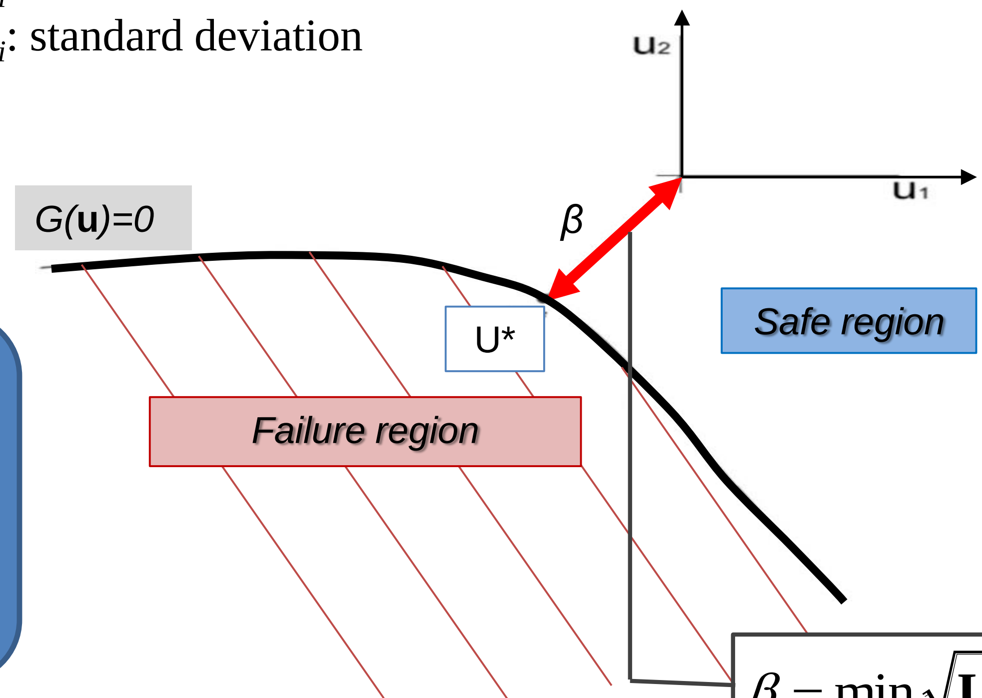
First Order Reliability Method (FORM)

Limit State Function: $G(\mathbf{x}) = R(\mathbf{x}) - S(\mathbf{x})$ *random variables* — *R*: resistance *S*: Load

Probability of Failure: $P_f = P[G(\mathbf{x}) < 0] = \Phi(-\beta)$ *reliability index*

x_1 and x_2 ⇒ $u_i = \dfrac{x_i - \mu_{x_i}}{\sigma_{x_i}}$

μ_{xi}: mean values
σ_{xi}: standard deviation

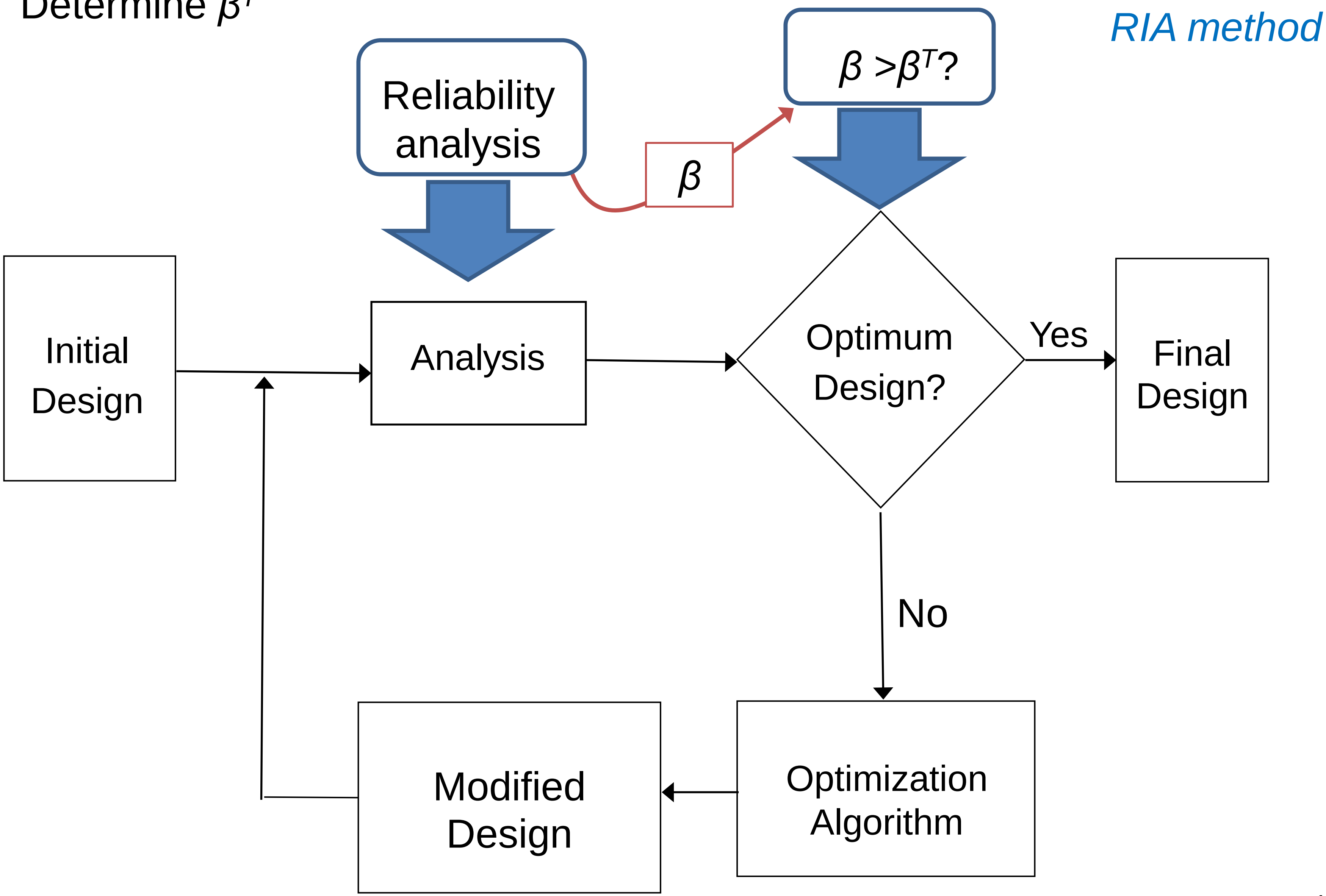


$$\beta = \min \sqrt{\mathbf{U}^T \cdot \mathbf{U}}$$

Eurocode for bridges (EN1990)
β=5.2 for 1 year period
β=4.3 for 50 years
β=3.8 for 100 years (P_f=7E-5)

Adding Reliability

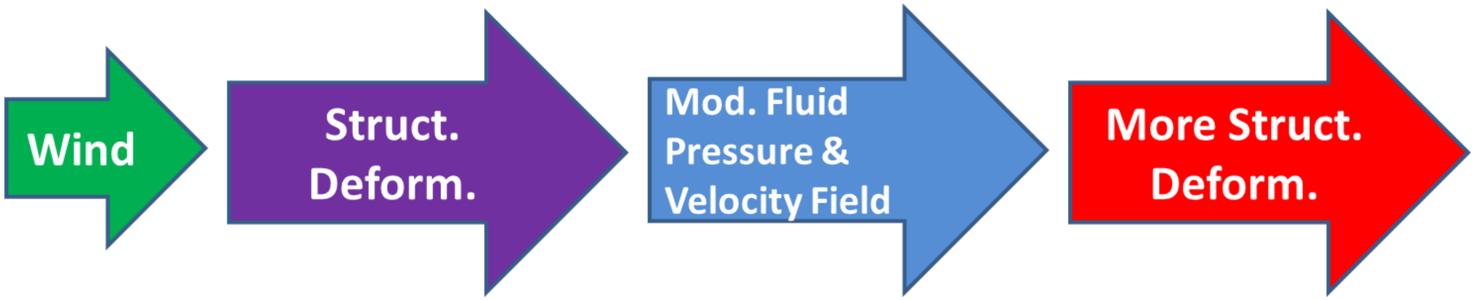
- Determine β^T



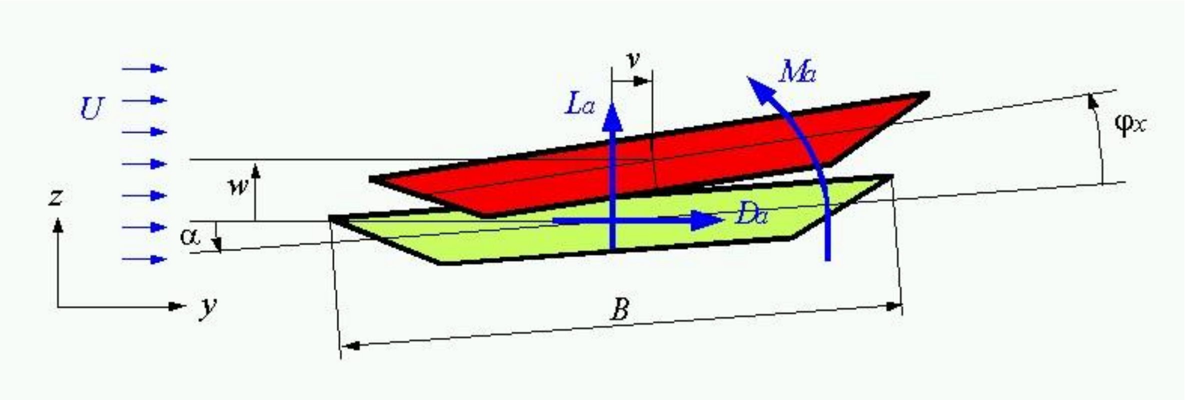
Reliability Analysis of flutter

What is Flutter?

- Aerodynamic instability of flexible structures
- Fluid structure interaction
- Coupling of modes
- Zero effective damping



Scanlan's Formulation



$$\mathbf{f}_a = \begin{Bmatrix} D_a \\ L_a \\ M_a \end{Bmatrix} = \frac{1}{2}\rho VKB \cdot \begin{pmatrix} P_1^* & P_5^* & BP_2^* \\ H_5^* & H_1^* & BH_2^* \\ BA_5^* & BA_1^* & B^2A_2^* \end{pmatrix} \begin{Bmatrix} \dot{v} \\ \dot{w} \\ \dot{\phi} \end{Bmatrix} + \frac{1}{2}\rho V^2 K^2 \cdot \begin{pmatrix} P_4^* & P_6^* & BP_3^* \\ H_6^* & H_4^* & BH_3^* \\ BA_6^* & BA_4^* & B^2A_3^* \end{pmatrix} \begin{Bmatrix} v \\ w \\ \phi \end{Bmatrix}$$

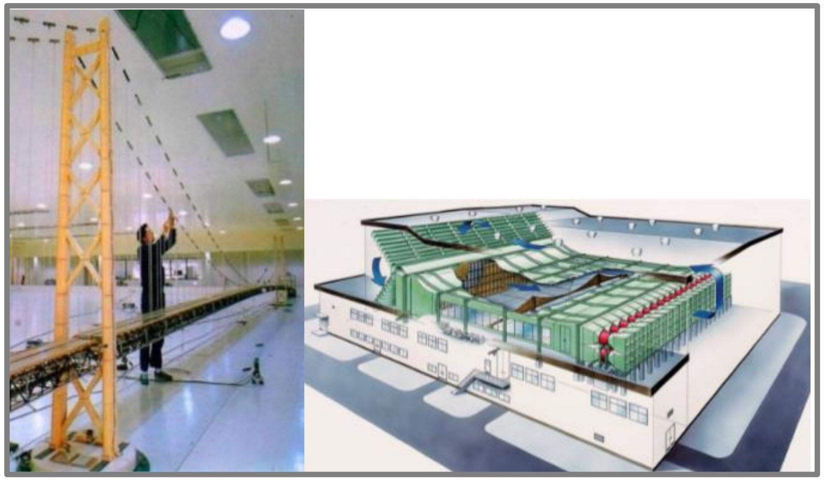
$$\mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} = \mathbf{f}_a = \mathbf{C}_a\mathbf{\dot{u}} + \mathbf{K}_a\mathbf{u} \quad \Longrightarrow \quad (\mathbf{A} - \mu\mathbf{I})\mathbf{w}_\mu e^{\mu t} = \mathbf{0}$$

$$\mu_j = \alpha_j \pm i\,\beta_j \begin{cases} \omega_j = \beta_j & \text{frequency} \\ \zeta_j = \dfrac{-\alpha_j}{\sqrt{\alpha_j^2 + \beta_j^2}} & \text{structural damping} \end{cases}$$

$\alpha_j=0 \rightarrow$ flutter instability

Methods for Flutter Analysis

Full Bridge Model Test

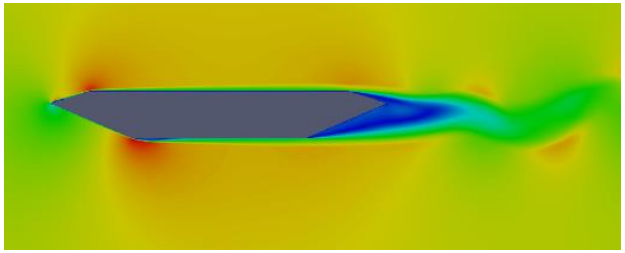


*Akashi bridge full model, PWRI

Hybrid Method

*Messina bridge sectional model, U. of Coruna

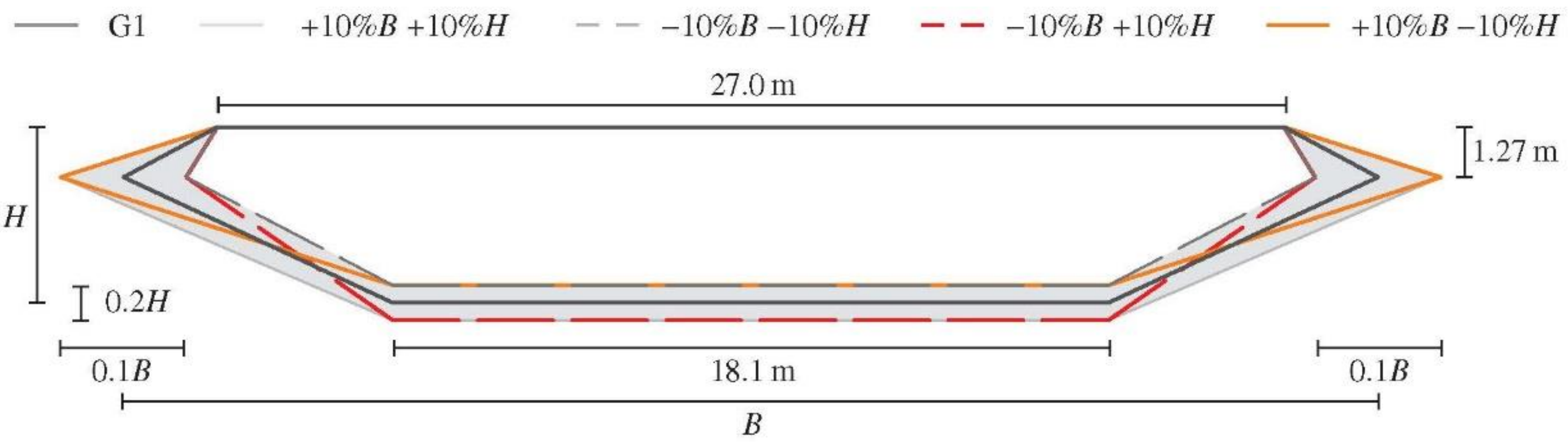
Fully Computational Method



+ Quasi Steady theory

Flutter analysis for Different Deck Shapes

1. Definition of the Deck Baseline Geometry and Design Range



Flutter analysis for Different Deck Shapes

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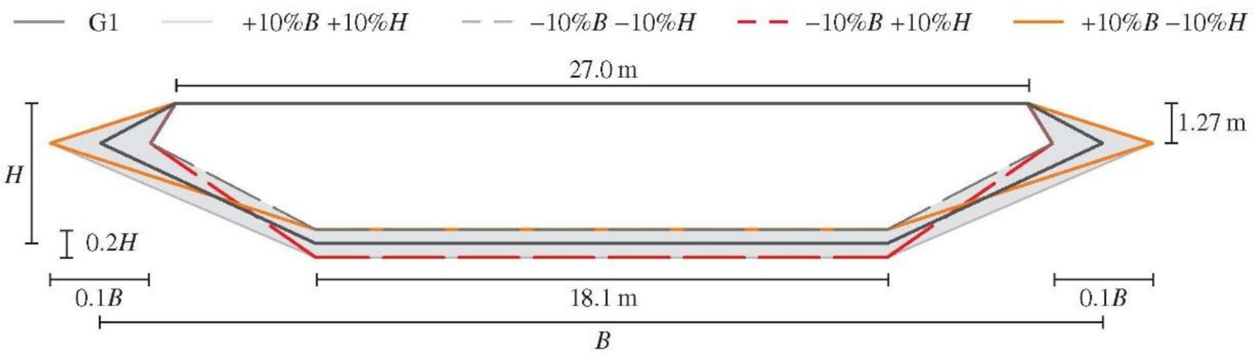
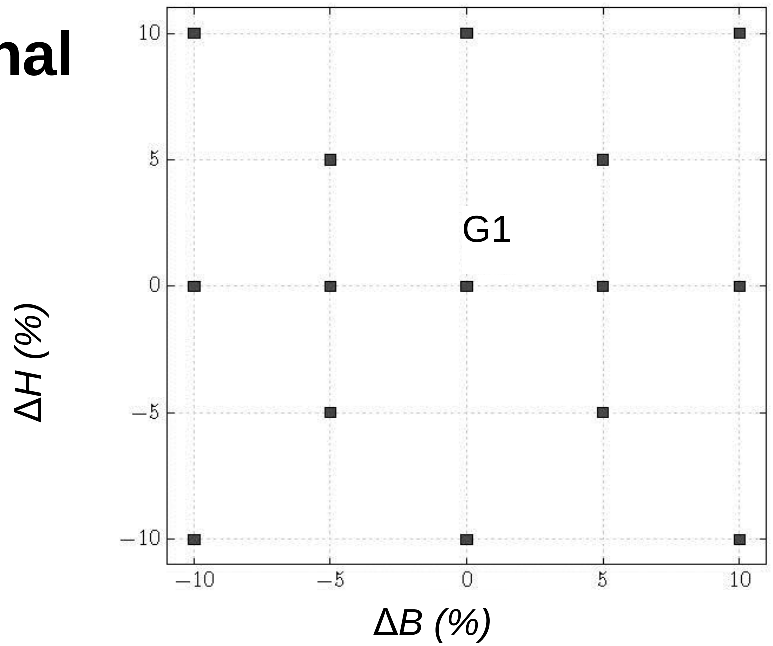


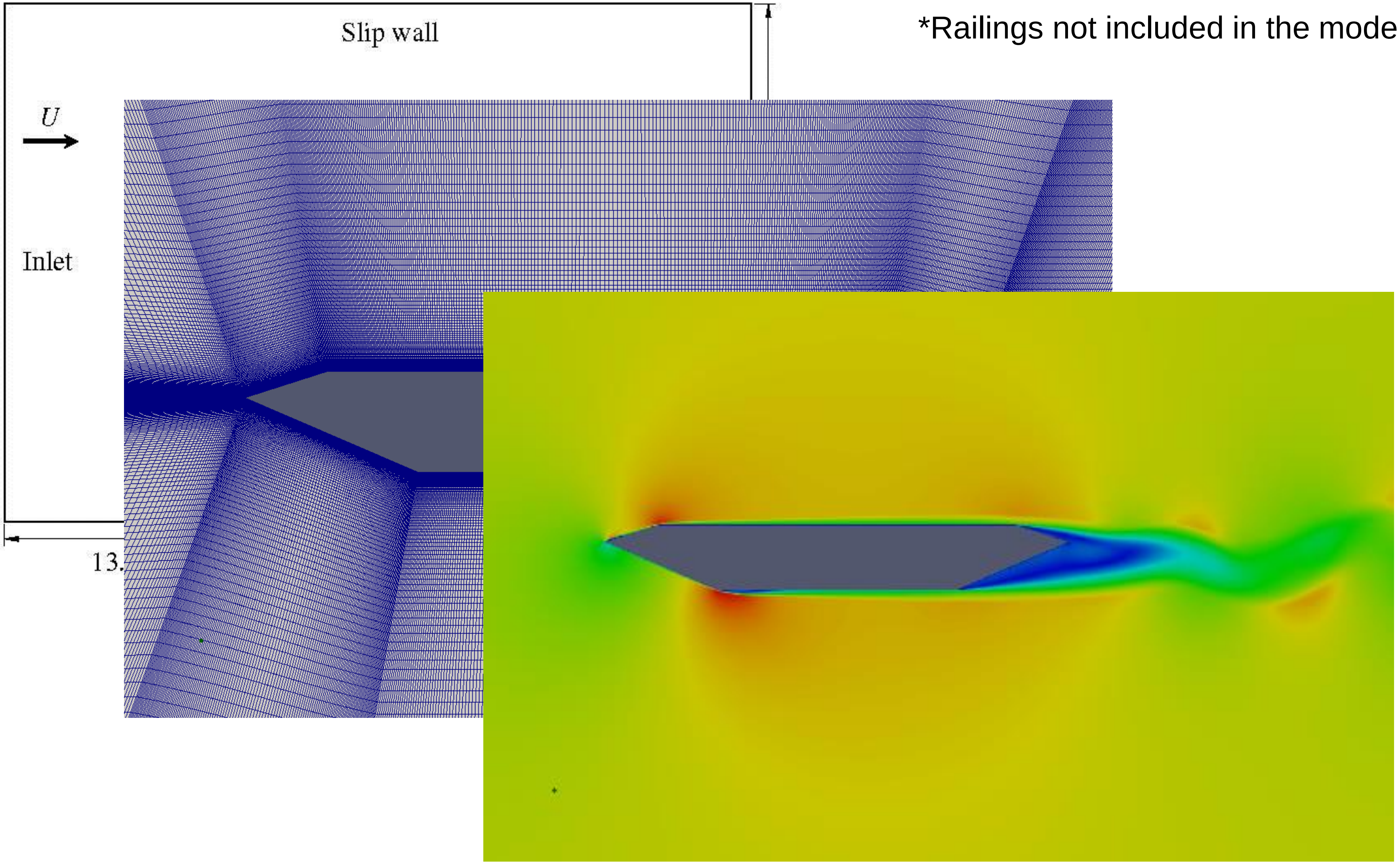
2. Sampling Plan of Computational Fluid Dynamics (CFD) Models



Flutter analysis for Different Deck Shapes

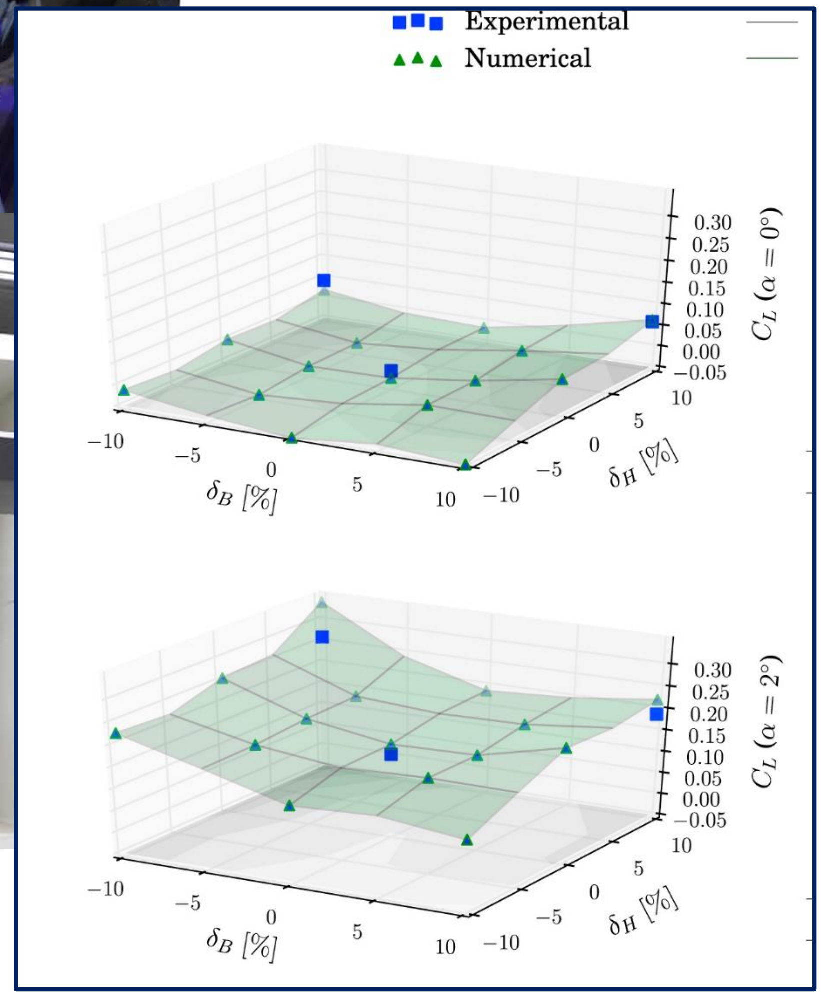
3. CFD Simulations by OpenFoam (kω-SST turbulence model)

*Railings not included in the model



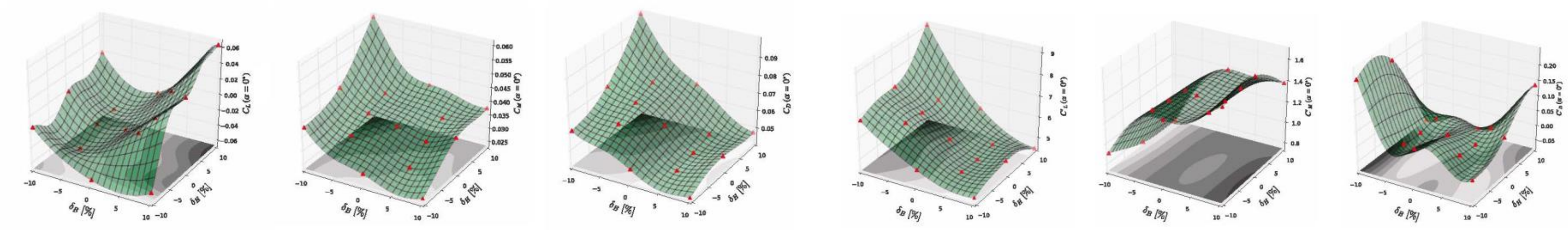
Flutter analysis for Different Deck Shapes

4. Wind Tunnel Test Validations



Flutter analysis for Different Deck Shapes

5. Kriging surrogate model construction



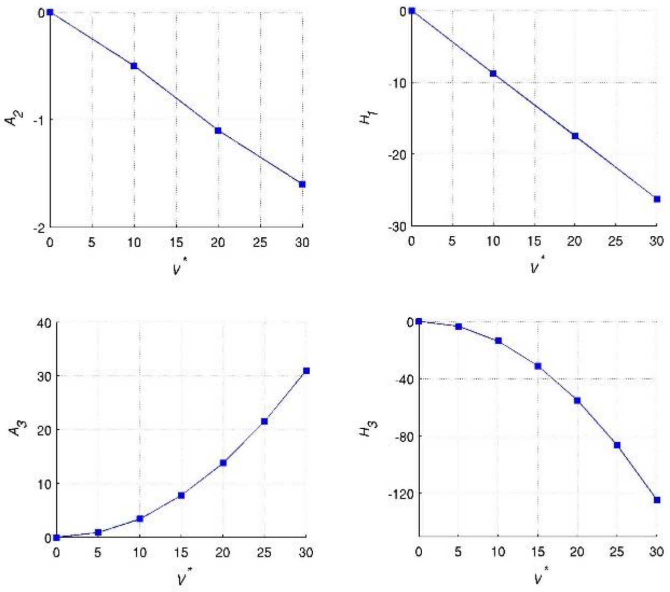
C_L C_D C_M C_L' C_D' C_M'

6. Quasi-steady formulation to define flutter derivatives

$$A_1^* = \frac{C'_{M,0^o}}{K} \qquad H_1^* = -\frac{C'_{L,0^o} + C_{D,0^o}}{K}$$

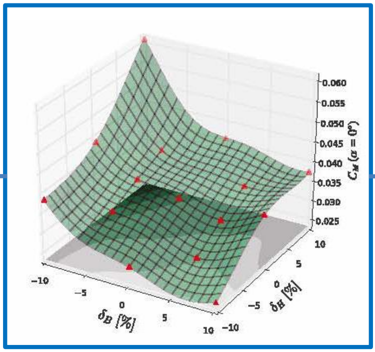
$$A_2^* = \frac{C'_{M,0^o}}{K} \cdot \mu_A \qquad H_2^* = \frac{C'_{L,0^o} + C_{D,0^o}}{K} \cdot \mu_H$$

$$A_3^* = -\frac{C'_{M,0^o}}{K^2} \qquad H_3^* = -\frac{C'_{L,0^o}}{K^2}$$

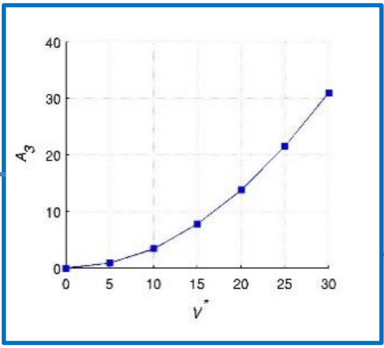


Flowchart: Flutter Analysis

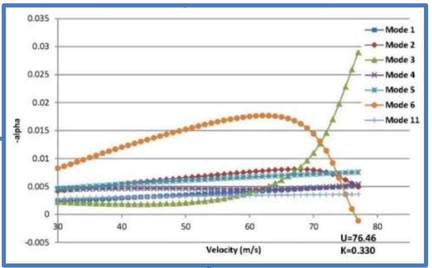
Define Deck Shape



QS

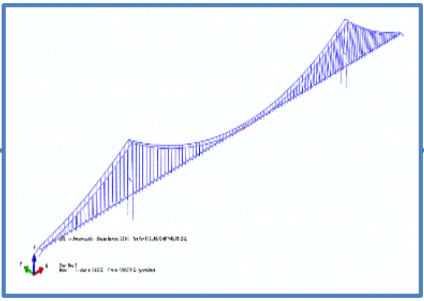


Multi-Modal Analysis

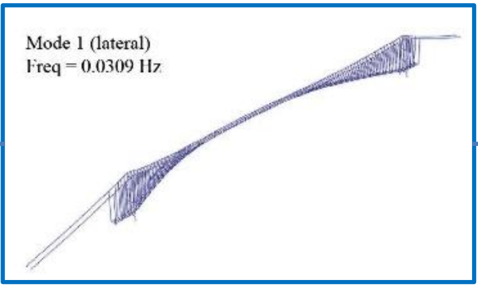


Vf

Modified FEM



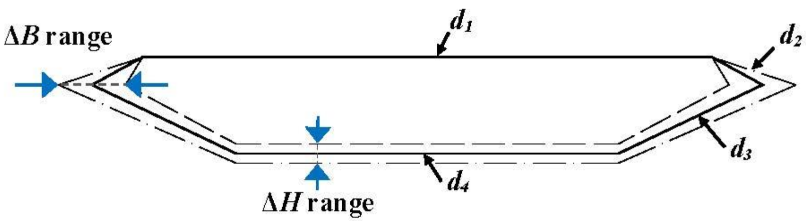
Φ *and ω*



RBDO Formulation: Shape & Size

Design Optimization

6 design variables:



Obj. Func.

Min: bridge deck volume

Constraints:

g1: probabilistic flutter

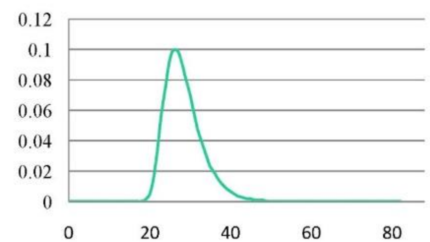
g2: side limits

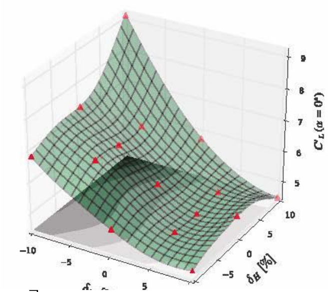
g3: deck max. vertical displacement under overload case

g4: max main cable stress

Reliability Analysis

7 random variables:





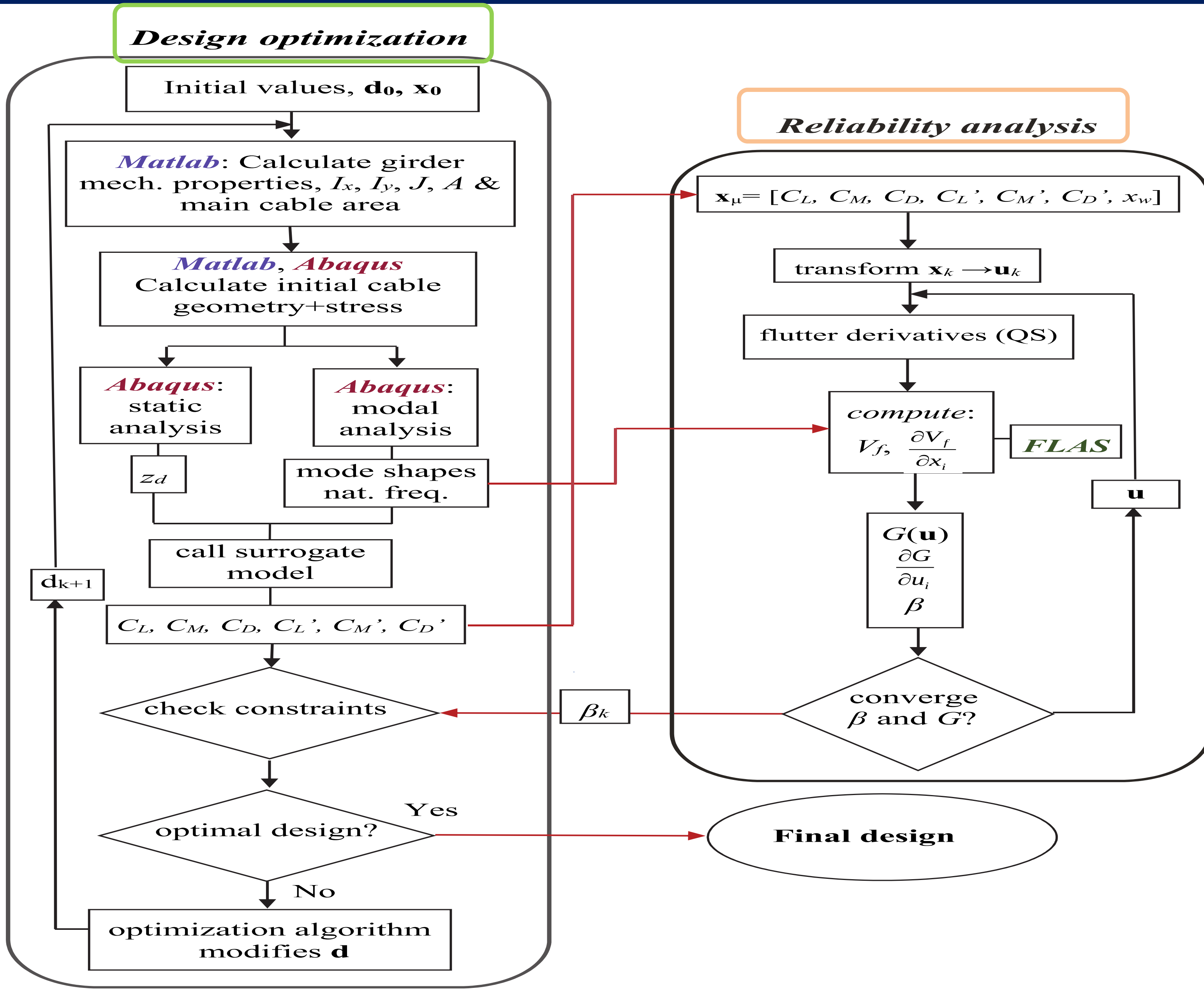
$$f_{x_w}(X_w) = \frac{1}{\beta}\exp\left(-\frac{X_w-\mu}{\beta}\right)\cdot\exp\left[-\exp\left(-\frac{X_w-\mu}{\beta}\right)\right]$$

$$G(\mathbf{x}) = V_f(x_i) - x_w$$

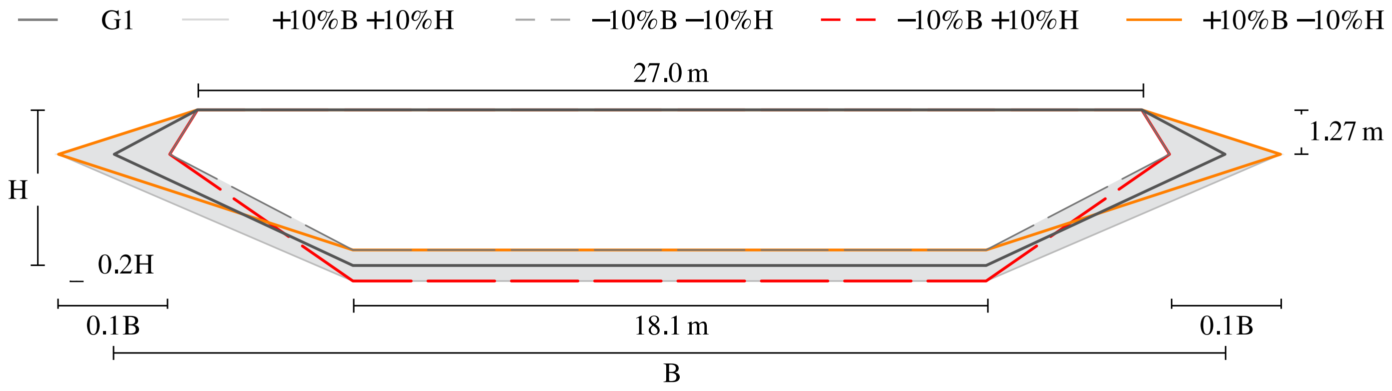
x_i: rand. variables of force coeff.

x_w: rand. variable of extreme wind

Flowchart: RBDO



Application Example: Great Belt East Bridge



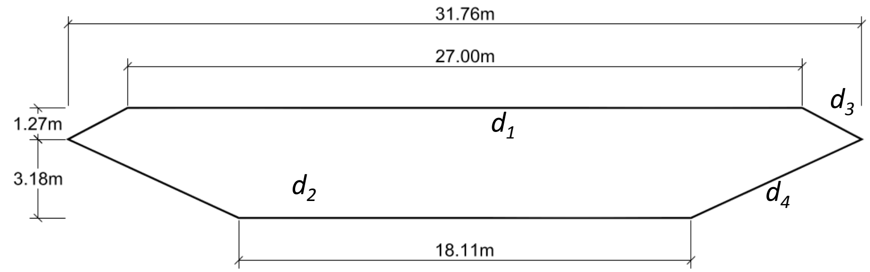
Scanlan's G1 Section

Flutter Analysis: Initial Design

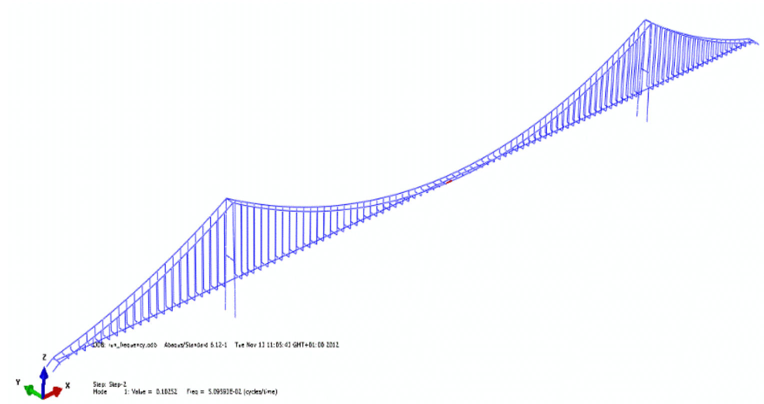
Mode Shapes and Frequencies

Mode	Type	Frequency (Hz)
2	VS	0.098
5	VS	0.131
11	LS	0.186
12	LS	0.195
13	LA	0.213
14	LS	0.213
15	VS	0.216
18	VS	0.249
19	LA	0.275
20	VS	0.282
21	TS/LS	0.285
22	VS	0.285
23	VA	0.286
24	TS/LS	0.290

V: vert. L: lat. T: tors. S: symm. A: asymm.



ΔH=0; ΔB=0
d=[12, 10, 10, 10] (mm)



V_f=78.2 m/s
K=0.47

Reliability Analysis of GB Bridge

Random variables of force coefficients

Limit State Function: $G = V_f(\mathbf{x}) - x_w$

Probability of Failure: $P_f = P[G(\mathbf{d},\mathbf{x}) \le 0]$

Random variables:

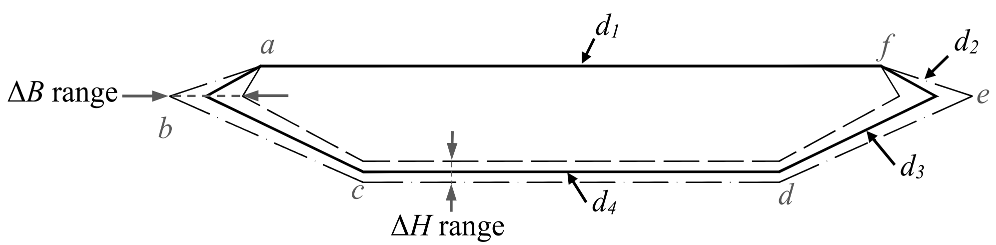
– *Case A*: Extreme Wind Velocity

– *Case B*: Force Coefficients, Derivatives, Extreme Wind Velocity

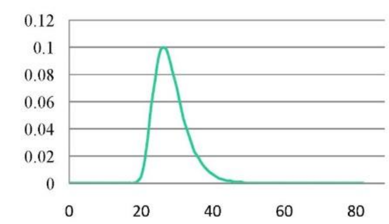
Case	random var.	*CV*	*β*	P_f	V_f (MPP)	V^*(MPP)
A	x_w	0.07	**12.01**	1.57E-33	78.20	13.22
B	x_w and x_i	0.2	**7.58**	1.73E-14	62.13	12.33

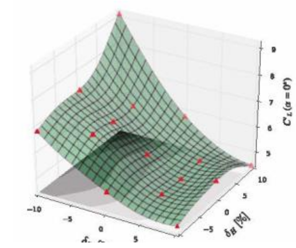
RBDO Formulation

6 Design Variables



7 Random Variables





$$f_{x_w}(X_w) = \frac{1}{\beta}\exp\left(-\frac{X_w-\mu}{\beta}\right)\cdot\exp\left[-\exp\left(-\frac{X_w-\mu}{\beta}\right)\right]$$ DS410

The RBDO formulation:

Minimize: Bridge Girder Volume

Subject to:

$g_1 : P\left[V_f(\mathbf{x}) - x_w \le 0\right] \le P_f$

$g_{2a} : -10\% \le \Delta H, \Delta B \le 10\%;\ \ g_{2b} : 5\text{ mm} \le t_i \le 25\text{ mm}$

$g_3 : z_d \le z_{max}\ \left(z_{max} = L/500; L = 1624m\right)$

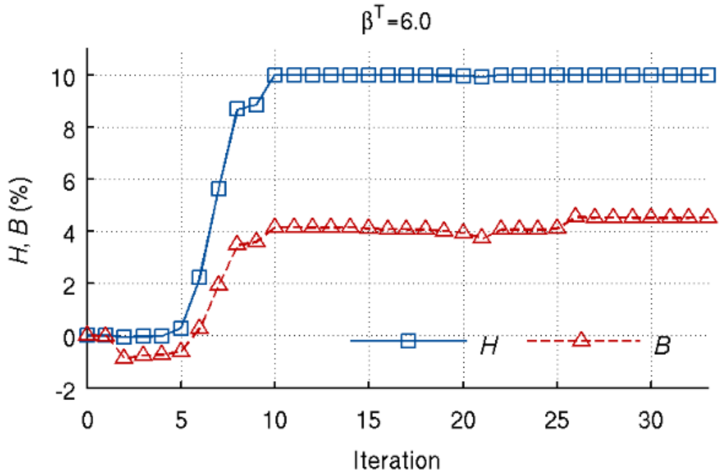
$g_4 : \sigma_c = 565\text{Mpa}$

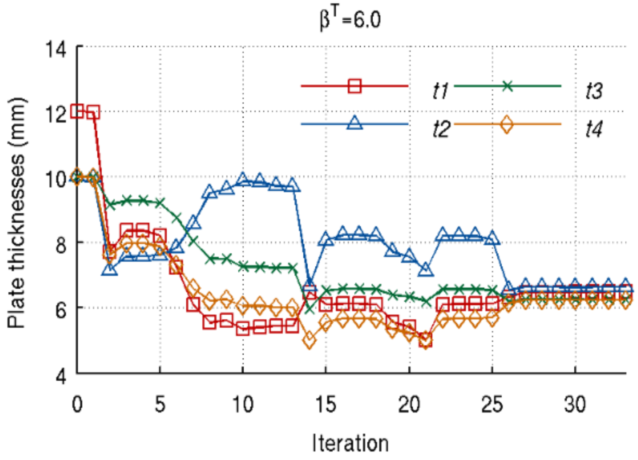
Initial Design
β=7.58

Target Reliabilities
β^T=6 and 8

RBDO Results

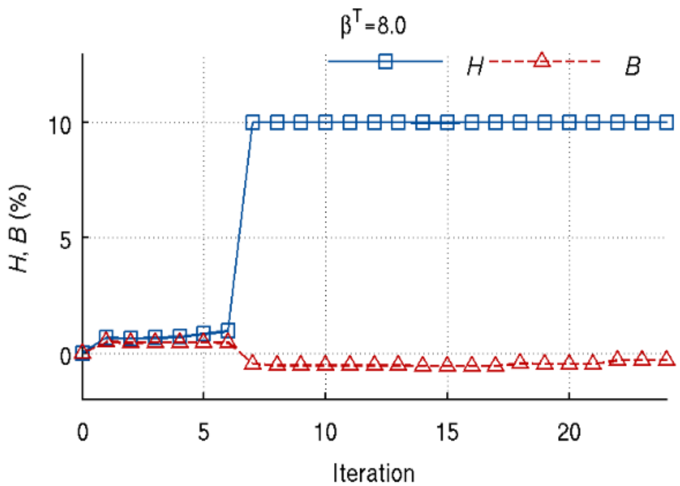
β^T=6.0

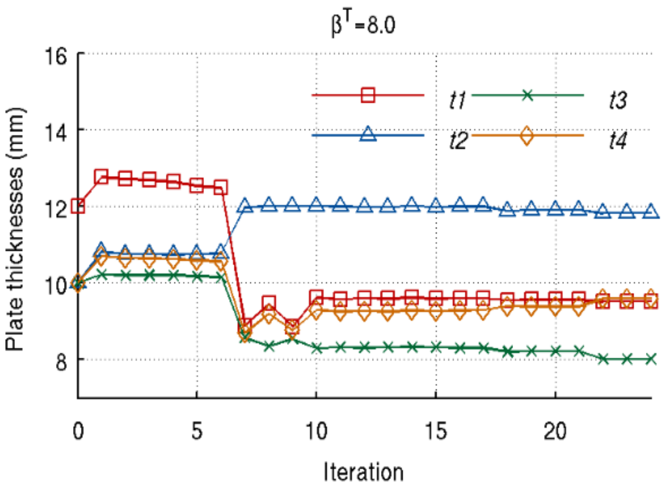




t1: top plate
t2: bottom plate
t3: upper side
t4: lower side

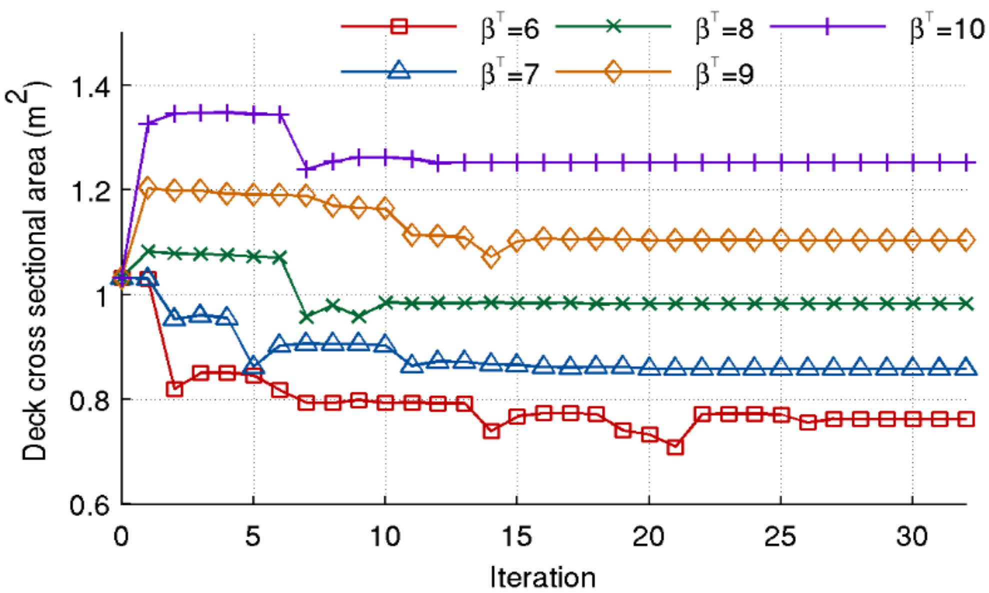
β^T=8.0





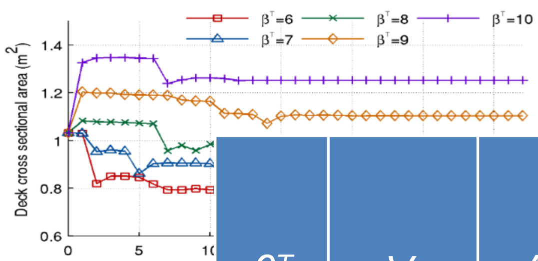
RBDO Results

Objective Function



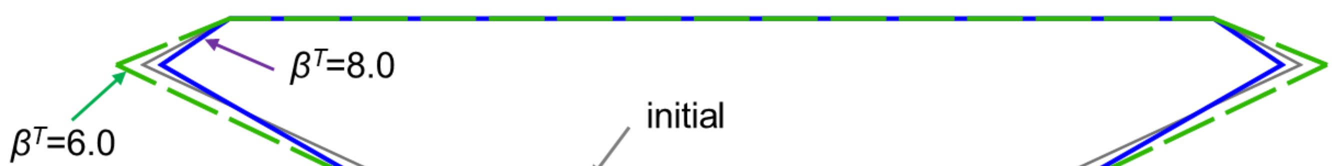
RBDO Results

Objective Function



RBDO Results Summary

β^T	V_f	ΔH	ΔB	d_1	d_2	d_3	d_4	% variation obj. func.
initial	78.2	0	0	12.0	10.0	10.0	10.0	-
6	69.45	10.00	4.51	6.48	6.63	6.24	6.24	-26.07
8	82.10	10.00	-0.32	9.51	11.82	8.02	9.58	-4.76



Summary

1. RBDO Provides Accurate & Competitive Optimum Design for Considering Uncertainty Explicitly.

2. Fully Numerical Approach of Flutter Velocity Computation Permits the Shape Optimization of Bridge Decks.

3. More Probabilistic Constraints in the Future Study *(aerodynamic instabilities, turbulence effects, traffic loads, temperature loads…)*

Thank you!

Thank you for your attention.

I hope you enjoyed the presentation